# WEEKLY TEST TARGET - JEE - TEST - 18
## SOLUTION Date 08-09-2019

## [PHYSICS]

1. **(c)** $V = V_0(1 + \gamma\Delta\theta)$

$$L^3 = L_0(1+\alpha_1\Delta\theta)L_0^2(1+\alpha_2\Delta\theta)^2$$
$$= L_0^3(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)^2$$

Since $L_0^3 = V_0$ and $L^3 = V$

Hence $1+\gamma\Delta\theta = (1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)^2$
$$\doteqdot (1+\alpha_1\Delta\theta)(1+2\alpha_2\Delta\theta)$$
$$\doteqdot (1+\alpha_1\Delta\theta+2\alpha_2\Delta\theta)$$

$\Rightarrow \quad \gamma = \alpha_1 + 2\alpha_2$

2. **(a)** As the coefficient of thermal expansion of the brass is greater than steel. Hence, the length of brass strip will be more than steel strip. Therefore, brass strip will be on convex side.

3. **(b)** As; $dl = \alpha l\, dT \quad \therefore \quad \int_l^{2l} \frac{dl}{l} = a\int_0^T T\, dT$

$$\ln 2 = a\frac{T^2}{2} \quad \therefore \quad T = \left[\frac{\ln 4}{a}\right]^{1/2}$$

4. **(c)** Strain developed:

$$\varepsilon = \alpha\Delta T = (12\times10^{-6})(50) = 6\times10^{-4}$$

Strain will be negative, as the rod is in a compressed state.

5. **(a)** For distance between $A$ and $F$ to remain constant, extension in $CD$ = extension in $AB$ + extension in $EF$

$\therefore \quad \Delta l_2 = 2\Delta l_1 \Rightarrow l_2\alpha_2\Delta\theta_2 = 2l_1\alpha_1\Delta\theta$

or $\quad \frac{l_1}{l_2} = \frac{\alpha_2}{2\alpha_1}$

6. **(c)** Strain $(\varepsilon) = \dfrac{\Delta l}{l} = \propto \Delta T = (10^{-5})(200) = 2 \times 10^{-3}$

Stress = $Y$ (strain)

Stress $= 10^{11} \times 2 \times 10^{-3} = 2 \times 10^{8}$ N/m$^2$

$\Rightarrow$ Required force = stress $\times$ Area

$= (2 \times 10^{8})(2 \times 10^{-6}) = 4 \times 10^{2} = 400$ N

$\therefore$ Mass to be attached $= \dfrac{400}{g} = 40$ kg

7. **(b)** The change in length of rod due to increase in temperature in absence of walls is

$$\Delta l = l\alpha\, \Delta T = 1000 \times 10^{-4} \times 20 \text{ mm}$$
$$= 2 \text{ mm}$$

But the rod can expand upto 1001 mm only.

At that temperature its natural length is = 1002 mm.

$\therefore$ compression = 1mm

$\therefore$ mechanical stress

$$= Y = \frac{\Delta l}{l} = 10^{11} \times \frac{1}{1000} = 10^{8} \text{ N/m}^2$$

8. **(a)** Given, decrease in temperature

$$(\Delta t) = 57 - 37 = 20°\text{C}$$

Coefficient of cubical expansion of copper

$$\gamma_{\text{Cu}} = 3\alpha_{\text{Cu}} = 5.1 \times 10^{-5}/°\text{C}$$

Let initial volume of the cavity be $V$ and its volume increases by $\Delta V$ due to increase in temperature.

$\therefore$ $\Delta V = \gamma V \Delta t$

$$\Rightarrow \frac{\Delta V}{V} = \gamma \Delta t$$

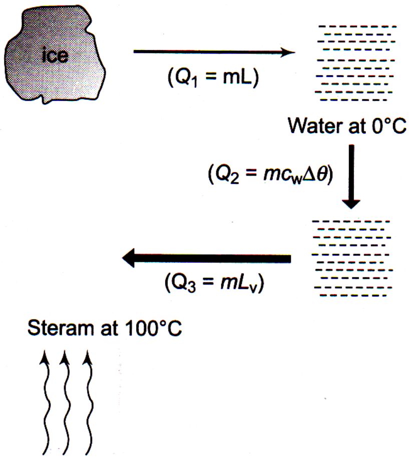
Thermal stress produced = $B \times$ Volumetric strain

$$= B \times \frac{\Delta V}{V} = B \times \gamma \Delta t$$

$$= 140 \times 10^{9} \times (1.5 \times 10^{-5} \times 20) = 42 \times 10^{6} \text{ N/m}^2$$

This is about $10^3$ times of atmospheric pressure.

9. **(c)** Ice (0°C) converts into steam (100°C) in following three steps.




Total heat required $Q = Q_1 + Q_2 + Q_3$

$= 5 \times 80 + 5 \times 1 \times (100 - 0) + 5 \times 540 = 3600$ cal

10. **(c)** $Q_1 = 10 \times 1 \times 10 = 100$ cal

$Q_2 = 10 \times 0.5(0 - (-20)) + 10 \times 80$

$= (100 + 800)$ cal $= 900$ cal.

As $Q_1 < Q_2$, so ice will not completely melt and final temperature = 0°C.

As heat given by water in cooling up to 0°C is only just sufficient to increase the temperature of ice from

11. **(a)** $W = JQ \Rightarrow \frac{1}{2}\left(\frac{1}{2}Mv^2\right) = J(m.c.\Delta\theta)$

$\Rightarrow \quad \frac{1}{4} \times 1 \times (50)^2 = 4.2[200 \times 0.105 \times \Delta\theta]$

$\Rightarrow \quad \Delta\theta = 7.1°C$

12. **(b)** $\frac{m_A c_A}{m_B c_B} = \frac{(4/3)\pi r_A^3 \rho_A c_A}{(4/3)\pi r_B^3 \rho_B c_B} = \left(\frac{r_A}{r_B}\right)^3 \frac{\rho_A c_A}{\rho_B c_B} =$

$\frac{m_A c_A}{m_B c_B} = \left(\frac{1}{2}\right)^3 \times \frac{2}{1} \times \left(\frac{1}{3}\right) = \frac{1}{12}$

13. **(a)** (i) Thermal energy

$TC = mc = \frac{Q}{\Delta T} = \frac{300}{45 - 25} = 15$ J/°C

(ii) Specific heat is nothing but thermal capacity per unit mass

$C = \frac{mc}{m} = \frac{15}{25 \times 10^{-3}} = 600$ J/kg-°C

14. **(a)** If heat is supplied at constant rate $P$, then $Q = P\Delta\tau$ and as during change of state $Q = mL$, so, $mL = P\Delta t$

i.e., $L = \left[\frac{P}{m}\right]\Delta t = \frac{P}{m}$ (length of line $AB$)

Hence $L_1 > L_2$

i.e., the ratio of latent heat of fusion of the two substances are in the ratio 3 : 4.

In the portion $OA$ the substance is in solid state and its temperature is changing.

$$\Delta Q = mC\Delta T \text{ and } \Delta Q = P\Delta t$$

So, $\frac{\Delta T}{\Delta t} = \frac{P}{mC}$ or slope $= \frac{P}{mS} = \left[\text{as } \frac{\Delta T}{\Delta t} = \text{slope}\right]$

Hence $C_1 < C_2$

15. **(c)** $x$ gm ice convert into $x$ gm water

$$\frac{x}{0.9} - x = 1 \Rightarrow x = \frac{0.9}{0.1} = 9$$

$\therefore$ $Q = 9 \times 80 = 720$ cal

16. **(a)** Water of mass $m_1 = 0.150$ kg is taken in the calorimeter at temperature $T_1 = 25°C$ is mixed with $N$ another known mass of pure water $m_2 = 0.400$ kg at a temperature $T_2 = 30°C$ and final temperature is found to be $T = 27°C$. Let $s_w$ and $W$ be the heat capacity of water and the water equivalent of the calorimeter.

Heat gained by (water + calorimeter) at temperature $T_1$ = Heat lost by water at temperature $T_2$

i.e., $m_1 s_w(T - T_1) + W s_w(T - T_1) = m_2 s_w(T_2 - T)$

or $W = m_2\left(\frac{T_2 - T}{T - T_1}\right) - m_1$

Hence, $W = 0.400\text{ kg}\left[\frac{30°C - 27°C}{27°C - 25°C}\right] - 0.150\text{ kg}$

$= 0.450$ kg

17. **(a)** Let '$m$' g be the required mass of water.

Then heat lost by hot water

$$Q_{lost} = (mg)\left(1\frac{\text{cal}}{\text{g-°C}}\right)(50-40)\text{°C} \qquad \text{(i)}$$

Heat gained by the calorimeter

$$Q_1 = \left[(1500\text{ g})\left(\frac{390}{4.2\times10^3}\frac{\text{cal}}{\text{g-°C}}\right)\right](40-25)\text{°C}$$

Heat gained by the water present in it

$$Q_2 = \left[(200\text{ g})\left(1\frac{\text{cal}}{\text{g-°C}}\right)\right](40-25)\text{°C}$$

Total heat gained by the calorimeter and the water present in it

$$Q_{gain} = Q_1 + Q_2 \qquad \text{(ii)}$$

From principle of calorimetry,

heat lost by hot body = heat gained by cold body

So, equating Eqs. (i) and (ii), and solving for $m$, we get

$$m = 508.93\text{ g}$$

18. **(c)** Rate of flow of heat $\frac{dQ}{dt}$ or $H$ is equal throughout the rod,

Temperature difference is given by

T.D. = (H) (Thermal Resistance)

or T.D. $\propto$ Thermal Resistance R

where $R = \frac{l}{KA}$ or $R \propto \frac{l}{A}$

Area across $CD$ is less. Therefore, T.D. across $CD$ will be more.

19. **(d)** By symmetry

$I_{AB} = I_{BC}$ and $I_{AD} = I_{DC}$

$\therefore$ No current in $BO$ and $OD$

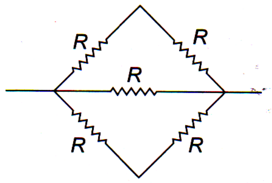
$\therefore$ $T_B = T_O = T_D$

20. **(c)** Initially effective resistance = $2R$. In parallel effective resistance = $\frac{R}{2}$. It has reduced by a factor of 1/4 so rate of heat transfer would be increased by a factor of 4, keeping other parameters same.

21. **(c)** $\frac{dQ}{dt}=\frac{KA\Delta T}{2l}=\frac{\Delta T}{\frac{2l}{KA}}=\frac{10}{120}$ J/sec.

New rate $\frac{d\dot{Q}}{dt}=\frac{\Delta T}{\frac{l}{2KA}}=\frac{40}{120}$ J/sec.;

So time taken is $t=\frac{20}{40}\times 120\text{ sec.}=60\text{ sec.}$




22. **(b)** In steady state $\left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer1}}=\left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer4}}$

$\Rightarrow\quad \frac{0.06\times A\times(30-25)}{1.5\times 10^{-2}}=\frac{0.10\times A\times \Delta T}{3.5\times 10^{-2}}$

$\Rightarrow\quad \Delta T=7°C$

$T_3=(-10+7)°C=-3°C$

23. **(a)** Area under given curve represents emissive power and emissive power $\propto T^4 \Rightarrow A\propto T^4$

$\Rightarrow\quad \frac{A_2}{A_1}=\frac{T_2^4}{T_1^4}=\frac{(273+327)^4}{(273+27)4}=\left(\frac{600}{300}\right)^4=\frac{16}{1}$

24. **(d)** Absolute temperatures of the black body corresponding to curve $P$ and $Q$ are in the inverse ratio of $\lambda_m$ (Wein's displacement law).

i.e., $\frac{T_P}{T_Q}=\frac{1987}{2980}$

Area under curves represent the total power radiated by a body and is proportional to the fourth power of absolute temperature (Stefan's law)

$\therefore\quad \frac{A_P}{A_Q}=\left(\frac{T_P}{T_Q}\right)^4=\frac{16}{81}$

25. **(d)** From Wien's law, $\lambda_m T=$ constant, where $T$ is the temperature of black body and $\lambda_m$ is the wavelength corresponding to maximum energy of emission. Energy distribution of black body radian is given below:

(i) $U_1$ and $U_2$ are not zero because a black emits nearly radiations of all wavelengths.

(ii) Since $U_1$ corresponding to lower wavelength, $U_3$ corresponds to higher wavelength and $U_2$ corresponds to medium wave length, hence $U_2 > U_1$.

26. **(c)** According to Stefan-Boltzmann law, the energy radiated per second through the surface of area $A$ is given by

$$E = \sigma A T^4$$

$\therefore \quad \dfrac{E_1}{E_2} = \dfrac{A_1}{A_2}\left(\dfrac{T_1}{T_2}\right)^4$

or $\quad 10000 = \dfrac{r_1^2}{r_2^2}\left(\dfrac{2000}{6000}\right)^4$

or $\quad \dfrac{r_1^2}{r_2^2} = (30)^4,$

or $\quad r_1 : r_2 = 900 : 1$

27. **(d)** $\lambda_1 T_1 = \lambda_2 T_2$

$\therefore \quad \dfrac{\lambda_1}{\lambda_2} = \dfrac{T_2}{T_1} = \dfrac{0.26}{0.15} = 2$

$\therefore \quad T_2 = 2T_1$

By Stefan's law, emissive power $E = \sigma T^4$

$$E_1 = \sigma T_1^4; \quad E_2 = \sigma T_2^4$$

$\therefore \quad \dfrac{E_1}{E_2} = \dfrac{\sigma T_1^4}{\sigma T_2^4} = \dfrac{T_1^4}{(2T_1)^4} = \dfrac{1}{16}$

28. **(a)** By Stefan's law, Rate of cooling, $H \propto (T^4 - T_0^4))$

$\therefore \quad \dfrac{H'}{H} = \dfrac{(900)^4 - (300)^4}{(600)^4 - (300)^4} = \dfrac{9^4 - 3^4}{6^4 - 3^4} = \dfrac{3^4(3^4 - 1)}{3^4(2^4 - 1)}$

$\therefore \quad H' = \dfrac{16}{3}H$

29. **(b)** According to Wien's law, $\lambda_m \propto \dfrac{1}{T}$

When temperature becomes $\dfrac{3}{2}$ times $\lambda_m$ becomes $\dfrac{2}{3}$ times

30. **(c)** $\lambda_0 T_0 = b$

$$P = CT_0^4$$

$$n\lambda_0 \cdot \frac{T_0}{n} = b$$

or $\quad \lambda_1 T_1 = b$

So, $\quad T_1 = T_0/n$

$$P = CT_1^4 = C(T_0/n)^4 = P/n^4$$

## [CHEMISTRY]

31. $MnO_4^{2-} + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O] \times 3$

$$\left.\begin{matrix} C_2O_4^{2-} \rightarrow 2CO_2 + 2e^- \\ Fe^{2+} \rightarrow Fe^{3+} + e^- \end{matrix}\right] \times 5$$

$3MnO_4^{2-} + 5[C_2O_4^{2-} + Fe^{2+}] + 24H^+ \rightarrow 10CO_2 + 5Fe^{3+} + 12H_2O$

$5Fe_2C_2O_4 \equiv 3MnO_4^{2-}$

$1Fe_2C_2O_4 = \frac{3}{5}MnO_4^{2-} = 0.6\ MnO_4^{2-}$

32. Higher the oxidation number of central aotm in oxoacid, higher is the acidic character.

33.

| $HIO_4$ | $H_3IO_5$ | $H_5IO_6$ |
|---|---|---|
| 1 + x – 8 = 0 | 3 + x – 10 = 0 | 5 + x – 12 = 0 |
| x = + 7 | x = + 7 | x = + 7 |

34. Oxidation number of $Br_2$ (element form) = 0
In $BrO_3^-$, O.N. of Br + 3(–2) = –1
⇒ O.N. of Br = +5

35. $PO_4^{-3}$ : x + 4 × (–2) = – 3
x = + 5
$SO_4^{2-}$ : x + 4 × (–2) = – 2
x = + 6
$Cr_2O_7^{2-}$ : 2x + 7(–2) = – 2
x = +6

36. $H_4P_2O_5$: 4 + 2x – 10 = 0 ⇒ x = + 3
$H_4P_2O_6$: 4 + 2x – 12 = 0 ⇒ x = + 4
$HP_2O_7$: 4 + 2x – 14 = 0 ⇒ x = + 5

37. Fluorine is the most electronegative element. If never shows +ve oxidation state.

38.

39. Lower the reduction potential, better the reducing power. Hence, the correct choice is 'd'

40. $\overset{(0)}{3Cl_2} + 6OH^- \rightarrow \overset{(-1)}{5Cl^-} + \overset{(+5)}{ClO_3^-} + 3H_2O$

41. Higher the reduction potential easier to gain electrons.
Lower the reduction potential easier the loss of electrons.

42. $FeCl_2$ and $SnCl_2$ do not react as both are reducing agents.

43.

44. In $SO_4^{2-}$, S is already in its highest oxidation state + 6

45.

46.

47. Calculations are done separately for N atom in $NH_4^+$ and $NO_3^-$.

48.

49. In $C_2O_4^{2-}$, C-atom has O.N. + 3. In $CO_2$, C-atom has O.N. +4. Hence, $C_2O_4^{2-}$ is reducing agent

50. In $NH_2$–$NH_2$, N-atom is in –2 oxidation state. 5 electrons are lost per atom.
$N^{2-} \rightarrow 5e^- + N^x$
– 2 = – 5 + x ⇒ x = + 3

51.

52.

53.

54. In this question, reduction potentials are int he order Li < Ba < Mg. Hence, the order of reducing power is Li > Ba > Mg.

55.

56. Higher the reduction potential, easier is the gain of electrons.

57.

58.

## [MATHEMATICS]

61. (a) Required distance $= \frac{7}{\sqrt{(12)^2+5^2}} = \frac{7}{13}$.

62. (c) Let $p$ be the length of the perpendicular from the vertex (2, –1) to the base $x+y=2$.

Then $p = \left|\frac{2-1-2}{\sqrt{1^2+1^2}}\right| = \frac{1}{\sqrt{2}}$

If '$a$' be the length of the side of triangle, then $p = a\sin 60^o \Rightarrow \frac{1}{\sqrt{2}} = \frac{a\sqrt{3}}{2} \Rightarrow a = \sqrt{\frac{2}{3}}$.

63. (a) The three lines are concurrent. if $\begin{vmatrix} 1 & 2 & -9 \\ 3 & 5 & -5 \\ a & b & -1 \end{vmatrix} = 0$

$\Rightarrow 35a - 22b + 1 = 0$

which is true if the line $35x - 22y + 1 = 0$ passes through $(a, b)$.

64. (c) Here $O$ is the point $(0,0)$. The line $2x+3y=12$ meets the $y$-axis at $B$ and so $B$ is the point (0,4). The equation of any line perpendicular to the line $2x+3y=12$ and passes through (5, 5) is $3x-2y=5$ ......(i)

The line (i) meets the $x$-axis at $C$ and so co-ordinates of $C$ are $\left(\frac{5}{3},0\right)$. Similarly the coordinates of $E$ are (3, 2) by solving the line $AB$ and (i). Thus $O(0, 0)$, $C\left(\frac{5}{3},0\right)$, $E(3,2)$ and $B$ (0, 4). Now the area of figure $OCEB$ = area of $\Delta OCE$ + area of $\Delta OEB = \frac{23}{3}$ *sq. units.*

65. (b) After first transformation, the point will be (1, 4) and therefore, final point is $(1+2,4)=(3,4)$.

66. (b) Suppose we rotate the coordinate axes in the anti clockwise direction through an angle $\alpha$. The equation of the line $L$ with respect to old axes is $\frac{x}{a}+\frac{y}{b}=1$. In this question replacing $x$ by $x\cos\alpha - y\sin\alpha$ and $y$ by $x\sin\alpha + y\cos\alpha$, the equation of the line with respect to new axes is $\frac{x\cos\alpha - y\sin\alpha}{a} + \frac{x\sin\alpha + y\cos\alpha}{b} = 1$

$\Rightarrow x\left(\frac{\cos\alpha}{a}+\frac{\sin\alpha}{b}\right) + y\left(\frac{\cos\alpha}{b}-\frac{\sin\alpha}{a}\right) = 1$ .....(i)

The intercepts made by (i) on the co-ordinate axes are given as $p$ and $q$.

Therefore $\frac{1}{p} = \frac{\cos\alpha}{a}+\frac{\sin\alpha}{b}$ and $\frac{1}{q} = \frac{\cos\alpha}{b}-\frac{\sin\alpha}{a}$

Squaring and adding, we get $\frac{1}{p^2}+\frac{1}{q^2} = \frac{1}{a^2}+\frac{1}{b^2}$.

67. (a) Let $Q(a,b)$ be the reflection of $P(4,-13)$ in the line $5x+y+6=0$.

Then the mid-point $R\left(\frac{a+4}{2},\frac{b-13}{2}\right)$ lies on $5x+y+6=0$.

$\therefore\ 5\left(\frac{a+4}{2}\right)+\frac{b-13}{2}+6=0 \Rightarrow 5a+b+19=0$ ......(i)

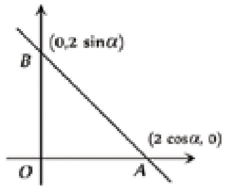
Also $PQ$ is perpendicular to $5x+y+6=0$.

Therefore $\frac{b+13}{a-4}\times\left(-\frac{5}{1}\right) = -1 \Rightarrow a-5b-69=0$ .....(ii)

Solving (i) and (ii), we get $a=-1$, $b=-14$.

68. (d) Two sides $x-3y=0$ and $3x+y=0$ of the given triangle are perpendicular to each other. Therefore its orthocentre is the point of intersection of $x-3y=0$ and $3x+y=0$ *i.e,* (0, 0). Clearly (0,0) satisfies $3x-4y=0$.

**69.** (a) $\Delta = \frac{1}{2}(2\sin\alpha . 2\cos\alpha) = \sin 2\alpha$




**70.** (a) $(h-3)^2 + (k+2)^2 = \left|\frac{5h-12k-13}{\sqrt{25+144}}\right|$.

Replace $(h,k)$ by $(x,y)$, we get

$13x^2 + 13y^2 - 83x + 64y + 182 = 0$, which is the required equation of the locus of the point.

**71.** (b) Let point be $(x_1, y_1)$, then according to the condition $\frac{3x_1 + 4y_1 - 11}{5} = -\left(\frac{12x_1 + 5y_1 + 2}{13}\right)$

Since the given lines are on opposite sides with respect to origin, hence the required locus is $99x + 77y - 133 = 0$

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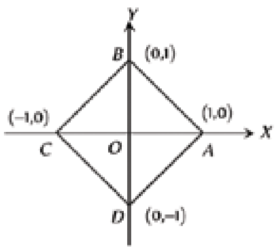
**72.** (a) Required locus of the point $(x,y)$ is the curve $|x| + |y| = 1$. If the point lies in the first quadrant, then $x > 0, y > 0$ and so $|x| + |y| = 1 \Rightarrow x + y = 1$, which is straight line *AB*. If the point $(x,y)$ lies in second quadrant then $x < 0$, $y > 0$ and so $|x| + |y| = 1 \Rightarrow -x + y = 1$




Similarly for third and fourth quadrant, the equations are $-x - y = 1$ and $x - y = 1$.
Hence the required locus is the curve consisting of the sides of the square *ABCD*.

**73.** (b) The equation of a line passing through the intersection of straight lines $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ and $\frac{x}{\beta} + \frac{y}{\alpha} = 1$ is

$$\left(\frac{x}{\alpha} + \frac{y}{\beta} - 1\right) + \lambda\left(\frac{x}{\beta} + \frac{y}{\alpha} - 1\right) = 0$$

or $$x\left(\frac{1}{\alpha} + \frac{\lambda}{\beta}\right) + y\left(\frac{1}{\beta} + \frac{\lambda}{\alpha}\right) - \lambda - 1 = 0$$

This meets the axes at

$$A\left(\frac{\lambda + 1}{\frac{1}{\alpha} + \frac{\lambda}{\beta}}, 0\right) \text{ and } B\left(0, \frac{\lambda + 1}{\frac{1}{\beta} + \frac{\lambda}{\alpha}}\right).$$

Let $(h, k)$ be the mid point of *AB*,

then $h = \frac{1}{2} . \frac{\lambda + 1}{\frac{1}{\alpha} + \frac{\lambda}{\beta}}, k = \frac{1}{2} . \frac{\lambda + 1}{\frac{1}{\beta} + \frac{\lambda}{\alpha}}$

Eliminating $\lambda$ from these two, we get

$$2hk(\alpha + \beta) = \alpha\beta(h + k).$$

$\therefore$ The locus of $(h,k)$ is $2xy(\alpha + \beta) = \alpha\beta(x + y)$.

Solving; locus of point $(x,y)$ is $6x + y - 32 = 0$.

74. (b) Equation of line passing through point (1, 1) is,

$y-1=m(x-1)$ ......(i)

Line (i) meets $x$-axis, so $y=0$

$\therefore \frac{-1}{m}=x-1 \Rightarrow x=1-\frac{1}{m}$

Line (i) meets $y$-axis, so $x=0$

$\therefore y-1=-m \Rightarrow y=1-m$

Let mid point of $AB$ be $(h, k)$,

Then $h=\frac{0+(1-(1/m))}{2}; k=\frac{0+(1-m)}{2}$

$m=\frac{1}{1-2h}$ ; $m=1-2k$

$\therefore 1-2k=\frac{1}{1-2h}$

$\Rightarrow 1-2k-2h+4hk=1 \Rightarrow -2h-2k+4hk=0$

Hence the Locus of mid point is, $x+y-2xy=0$.

75. (c) According to question, $x_1=\frac{2+4+x}{3} \Rightarrow x=3x_1-6$

$y_1=\frac{5-11+y}{3} \Rightarrow y=3y_1+6$

$\therefore 9(3x_1-6)+7(3y_1+6)+4=0$

Hence locus is $27x+21y-8=0$, which is parallel to $9x+7y+4=0$.

76. (a) $-6.10k+\frac{11.1.31}{4}-6\left(\frac{31}{2}\right)^2+10\left(\frac{1}{2}\right)^2-k\left(\frac{11}{2}\right)^2=0$

$\Rightarrow -k\frac{361}{4}=\frac{5415}{4} \Rightarrow k=-15$.

77. (d) Here $m_1+m_2=\frac{-2h}{b}$ .....(i)

and $m_1m_2=\frac{a}{b}$ .....(ii)

Also, given that $4ab=3h^2$. Now we have to find $\frac{m_1}{m_2}$,

therefore with the help of (i) and (ii), we get

$(m_1-m_2)^2=\frac{4h^2-4ab}{b^2}=\frac{4h^2-3h^2}{b^2}=\frac{h^2}{b^2}$

$\Rightarrow m_1-m_2=\frac{h}{b}$ .....(iii)

Now on solving (i) and (iii), we get $m_1=\frac{-h}{2b}$ and $m_2=\frac{-3h}{2b}$; $\therefore m_1:m_2=1:3$.

78. (d) If it represents two coincident straight lines, then the condition $h^2-ab=0$ should apply as angle between them would be zero. Hence $h^2-4=0$ or $h=\pm 2$.

79. (a) Equation is $2x^2-xy-6y^2+7x+21y-15=0$

Therefore, equation of lines parallel to given lines and passes through origin is homogeneous 2nd degree equation *i.e.*, $2x^2-xy-6y^2=0$.

80. (c) Here equation is $2x^2+4xy-py^2+4x+qy+1=0$.

The lines are perpendicular, if $a+b=0$

$\Rightarrow 2-p=0 \Rightarrow p=2$

and it will represent two lines, if

$abc+2fgh-af^2-bg^2-ch^2=0$

$\Rightarrow 2(-2)(1)+2\left(\frac{q}{2}\right)(2)(2)-2\left(\frac{q}{2}\right)^2+2(2)^2-1(2)^2=0$

$\Rightarrow q^2-8q=0 \Rightarrow q=0$ or 8.

81. (b) $m_1 = 3,\ m_2 = -\frac{1}{3}$. Hence, the lines are $y = 3x, y = -\frac{1}{3}x$.
Multiplying both the lines, we get
$(y-3x)(3y+x) = 0 \Rightarrow 3x^2 + 8xy - 3y^2 = 0.$

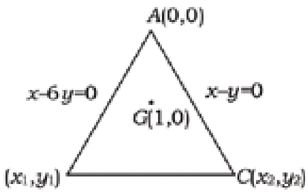
82. (b) The equation of lines represented by the equation $3x^2 - 8xy + 5y^2 = 0$ are $3x - 5y = 0$ and $x - y = 0$. Therefore, equation of lines passing through (1,2) and perpendicular to given lines are $x + y - 3 = 0$ and $5x + 3y - 11 = 0$.

83. (a) Applying the condition, $4\lambda h^2 = ab(1+\lambda)^2$
Here $\lambda = 2$, therefore
$4 \times 2 \times \left(\frac{h}{2}\right)^2 = 1 \times 2(1+2)^2 \Rightarrow h^2 = 9 \Rightarrow h = \pm 3$.

84. (a) $\tan\alpha \tan\beta = m_1 m_2 = \frac{a}{b} = -\frac{6}{7}$.

85. (c) We know that $m_1 - m_2 = \sqrt{(m_1+m_2)^2 - 4m_1m_2}$
$= \sqrt{\left(\frac{2\tan\theta}{\sin^2\theta}\right)^2 - 4\left(\frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}\right)}$
$= \sqrt{\frac{4\tan^2\theta}{\sin^4\theta} - 4(\sec^2\theta \operatorname{cosec}^2\theta - 1)} = 2$.

86. (d) Given two lines are $x - 6y = 0$ and $x - y = 0$.




We know $\frac{0 + x_1 + x_2}{3} = 1$
$\Rightarrow x_1 + x_2 = 3$ .....(i)
and $y_1 + y_2 = 0$ .....(ii)
Also $x_1 - 6y_1 = 0$ .....(iii)
$x_2 - y_2 = 0$ .....(iv)
[Since the point $(x_1, y_1)$ and $(x_2, y_2)$ lies on the lines $AB$ and $AC$ respectively]
Now on solving, the coordinates of $B$ and $C$ are $\left(\frac{18}{5}, \frac{3}{5}\right)$ and $\left(\frac{-3}{5}, \frac{-3}{5}\right)$ respectively.
Hence the equation of third side *i.e.*, $BC$ is $2x - 7y - 3 = 0$.

87. (d) $m_1 = \tan\alpha$ and $m_2 = \tan\beta$
$\Rightarrow \cot\alpha = \frac{1}{m_1}$ and $\cot\beta = \frac{1}{m_2}$
Hence, $\cot^2\alpha + \cot^2\beta = \frac{1}{m_1^2} + \frac{1}{m_2^2} = \frac{m_1^2 + m_2^2}{(m_1m_2)^2}$
$= \frac{(m_1+m_2)^2 - 2m_1m_2}{(m_1m_2)^2} = \frac{(3)^2 - 2(2)}{(2)^2} = \frac{5}{4}$

88. (d) $m_1 + m_2 = \frac{2\tan A}{-1} = 4$
$\Rightarrow \tan A = -2 \Rightarrow \angle A = \tan^{-1}(-2)$

89. (c) Distance $= 2\sqrt{\frac{g^2 - ac}{a(a+b)}} = 2\sqrt{\frac{4-1}{1(1+2)}} = 2$.

90. (c) Applying the formula, the distance between them is $\left|2\sqrt{\frac{(k^2/4) - 0}{1.(1+4)}}\right| = \left|\frac{k}{\sqrt{5}}\right|$,
$\therefore \left|\frac{k}{\sqrt{5}}\right| = 3 \Rightarrow k = \pm 3\sqrt{5}$.